Effect of beam ellipticity on self-mode locking in lasers

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In the cavity of a self-mode-locked (Kerr-lens mode-locked) laser, a noncircular beam experiences a nonlinear coupling between beam parameters in the x and y planes. This coupling produces a significant change in beam radii throughout the cavity and may result in more than one TEM_{00} cavity mode. The dramatic nature of this effect is demonstrated with the Ti:sapphire laser as an example.

Self-mode-locking is a common way of generating very short laser pulses.[1,2] It occurs when a beam self-focuses and narrows as it passes through a nonlinear intracavity material, thus expanding or contracting at other locations of the laser cavity.[3] Since self-focusing is an intensity-dependent process, at any point in the cavity an intense (mode-locked) laser beam has a different size from a weak (cw) laser beam. If cavity elements are arranged correctly, the high-intensity beam may experience lower diffraction losses, e.g., through slits or around prisms, than the low-intensity beam. The high-intensity beam may also experience higher gain in the active medium through better mode matching with the pump beam. These conditions tend to favor mode locking over cw operation. In practice, self-mode-locking is usually initiated by a mechanical perturbation.

In the literature, researchers have presented various methods for calculating the intensity-dependent beam radii in a laser cavity,[4–6] but none of these methods treated elliptical beams of the sort found in many lasers (e.g., lasers with Z- or X-shaped cavities). We show that elliptical beams are subject to the important effect of nonlinear x–y coupling, whereby the radius of the sagittal beam influences the index of refraction experienced by the tangential beam and vice versa. When we include this effect, the calculated radii differ dramatically from those found when the effect is excluded. We also reach completely different conclusions about where apertures should be placed to cause self-mode-locking or even whether self-mode-locking is possible for a particular arrangement of cavity elements.

We use an *ABCD* matrix method for calculating the beam radii throughout the laser cavity. This method is less exact than the two-dimensional split-step Fourier-transform method,[7] but it is computationally much faster. For high intracavity powers (near the power for catastrophic self-focusing), the more accurate Fourier-transform method should be used. We can further improve cavity calculations by including the effect of gain guiding,[4] which is neglected here.

In order to use *ABCD* matrix methods, we use a quadratic approximation of the Gaussian intensity to describe the change in the index of refraction,

$$\Delta n = n_2 I = n_2 \frac{2P}{\pi w_x w_y} \exp\left(-2\frac{x^2}{{w_x}^2} - 2\frac{y^2}{{w_x}^2}\right) \cong n_2 \frac{2P}{\pi w_x w_y}\left(b\frac{2x^2}{a_x {w_x}^2} - \frac{2y^2}{a_y {w_y}^2}\right), \quad (1)$$

where $I(x,y)$ is the beam intensity and $P = \iint I(x,y)\mathrm{d}x\mathrm{d}y$ is the beam power. If we set $y = 0$ in Eq. (1), we find that the index of refraction at a point in the x–z plane is dependent on not only the beam radius in the x direction (w_x) but also on the beam radius in the y direction (w_y). This is the origin of the nonlinear x–y coupling.

In relation (1), the parameters a_x, a_y, and b are found by minimization of the mean-square error.[6] We find the mean-square error by squaring the difference between the Gaussian and quadratic functions, weighting the result by the Gaussian function, and then integrating over the x–y plane.) We find that $a_x = a_y = a = 4$ and $b = 3/4$. The obvious value of $a = a_x = a_y = 1$, obtained with the Taylor expansion about $x = 0$, $y = 0$, is valid only near the beam center and, as we will see later, gives an effective intracavity power that is too large by a factor of 4. Another suggestion that was made for the value of a is based on Z-scan calculations.[5,8] This method gives $3.77 \le a_x, a_y \le 6.4$, with the particular values of a_x and a_y selected according to the size of apertures present in the laser cavity. For realistic aperture sizes, however, we expect that $a_x \cong a_y \cong 4$.

A quantity that we need in the equations that follow is the critical power, defined as the power at which a Gaussian beam eventually undergoes catastrophic self-focusing[9]:

$$P_{\mathrm{crit}} = aP_1 = a\frac{{\lambda_0}^2}{8\pi n_0 n_2}. \quad (2)$$

Within the quadratic approximation that we have made, $P_{\mathrm{crit}} = 4P_1$ because $a = 4$. Accurate numerical calculations made without the quadratic approximation show the critical power to be $3.77P_1$.

The method that we use in our calculations of beam radii is a generalization of the method presented in Ref. 6. For the case of a circular beam,

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this method provides a solution to the paraxial wave equation (with Kerr nonlinearity) that is exact within the quadratic approximation. A single *ABCD* matrix (which we call the self-focusing *ABCD* matrix) can therefore be used to calculate the radius of a propagating beam at any point in a Kerr medium, however long.[6] For the case of an elliptical beam, however, the nonlinear medium must be broken into segments to reflect changes in beam ellipticity that occur during propagation. For a given segment of thickness d, the self-focusing *ABCD* matrix is

$$M_i = \sqrt{1-\gamma_i}\begin{bmatrix} 1 & d/n_0 \\ -n_0\gamma_i d(1-\gamma_i) & 1 \end{bmatrix}, \quad i = x, y, \tag{3}$$

where

$$\gamma_x = [r(z)P/P_{\text{crit}}][1 + (n_0\pi w_{1x}^2/d\lambda_0)^2(1 + d/R_{1x})^2]^{-1},$$

$$\gamma_y = [P/r(z)P_{\text{crit}}][1 + (n_0\pi w_{1y}^2/d\lambda_0)^2(1 + d/R_{1y})^2]^{-1},$$

$$r(z) = w_x(z)w_y(z).$$

In these expressions, R_{1i} and w_{1i} (with $i = x, y$) are the radius of curvature and the spot size at the start of the segment. In practice, we evaluate $r(z)$ at the segment center. At each segment, we perform the matrix calculations in both the x and y directions before proceeding to the next segment. This ensures that $r(z)$ reflects the current values of $w_x(z)$ and $w_y(z)$.

We can apply *ABCD* matrix methods to the problem of an elliptical beam propagating in a Kerr medium without including the effect of nonlinear x–y coupling by simply setting $r(z) = 1$. This has the effect of uncoupling the radii in the x and y directions, i.e., of making them independent of one another. We will show, however, that this simplification can give results that are very far off.

In addition to the method above, two other methods of calculation have appeared in the literature, and both may be generalized to account for beam ellipticity. The first method is based on the application of quadratic ducts.[4,10] Compared with the method of Eq. (3), however, this method requires more segments in order to achieve the same accuracy. The second method is based on a change of variables.[5] Before entering the nonlinear medium, we expand the beam; $W_x^2 = w_x^2/[1 - r(z)P/P_{\text{crit}}]^{1/2}$, $W_y^2 = w_y^2/[1 - P/r(z)P_{\text{crit}}]^{1/2}$. Before leaving the nonlinear medium, we shrink the beam; $w_x^2 = W_x^2[1 - r(z)P/P_{\text{crit}}]^{1/2}$, $w_y^2 = W_y^2[1 - P/r(z)P_{\text{crit}}]^{1/2}$. This method is valid only when these roots are real [when $P < \min(r, 1/r)P_{\text{crit}}$], whereas the method of Eq. (3) is accurate for all powers P. We have checked the results obtained in calculations made by the three methods described above and find that all give the same results if the nonlinear medium is divided into enough segments. We also find that the error in the calculated beam radius when any of the above methods is used varies approximately as $1/n^2$, where n is the number of segments.

For the case of an elliptical beam, we cannot calculate the characteristics of the cavity mode by using a formula. Instead, we guess at the values of the beam radii w_x and w_y at an end mirror, and then we propagate the beam back and forth inside the cavity. After each round trip, we apply some damping to the beam, which is to say that we divide the radii of curvature in the x and y directions by $1 - \delta$, where the damping factor δ is a number between zero and 1. Application of a small damping factor results in ringing, whereas application of a large damping factor results in a slow but steady convergence. A value that is intermediate between the extremes produces the fastest convergence.

After we find a solution, we test that it is stable against perturbation[11] by slightly changing the radii w_x and w_y at an end mirror, and then we propagate the beam back and forth in the cavity without any damping. A beam that is unstable against perturbation will eventually diverge.

We now consider how nonlinear x–y coupling affects beam radii in two different Z-shaped cavities, one asymmetric and one symmetric, using parameters typical of a Ti:sapphire laser. Figure 1(a) shows the asymmetric cavity, and Fig. 1(b) shows the sagittal (y) radii for this cavity for an intracavity power of $P_{\text{crit}}/2$. When nonlinear x–y coupling is included, calculations show the mode-locked sagittal beam to be smaller than the cw sagittal beam on the right-hand side of the cavity. In this case, a horizontal slit placed at the right edge of the cavity would encourage mode locking by causing more loss for the cw beam

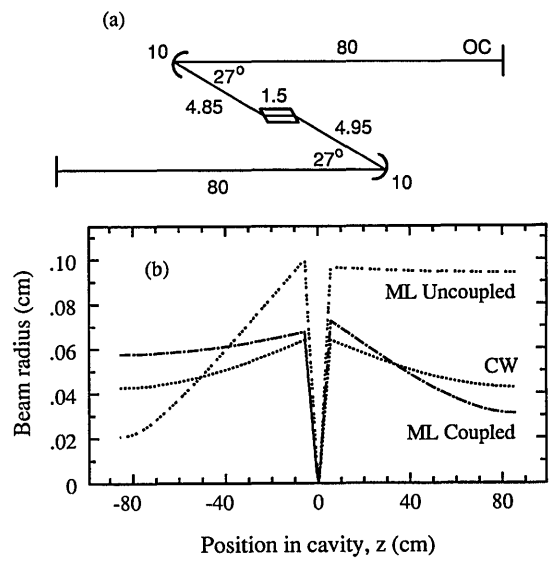

Fig. 1. (a) Asymmetric Z cavity. All distances are in centimeters. OC, output coupler. (b) Beam radii in the sagittal (y) plane as a function of position in the laser cavity by use of the following parameters typical of a Ti:sapphire laser: $\lambda_0 = 800$ nm, $n_0 = 1.76$, $a = 4$, $n_2 = 3 \times 10^{-16}$ cm^2/W. The position $z = 0$ is at the center of the crystal; positions $z < 0$ are on the side containing the output coupler (the left side). Three beams are calculated; a low-intensity beam without nonlinearity (CW), a mode-locked beam with nonlinear x–y coupling (ML Uncoupled). Because of nonlinear x–y coupling, a horizontal slit placed at the right edge of the cavity would encourage mode locking by causing more loss for the cw beam than for the mode-locked beam.

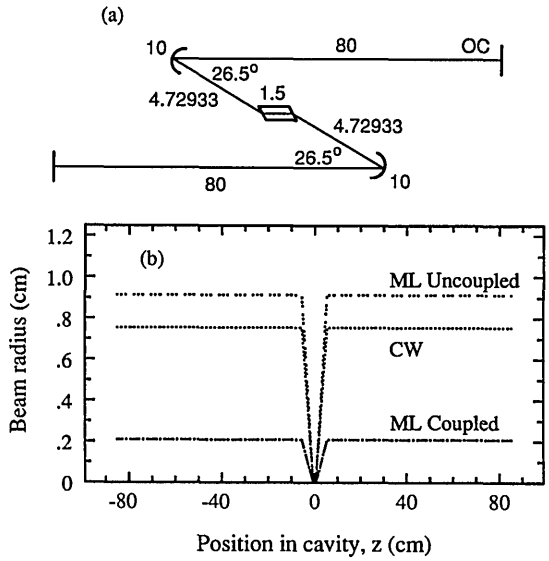

Fig. 2. (a) Symmetric Z cavity. (b) Beam radii in the tangential (x) plane as a function of position in the laser cavity by use of the same parameters as in Fig. 1. Because of nonlinear $x-y$ coupling, a vertical slit or the edge of a prism placed anywhere in the cavity would encourage mode locking.

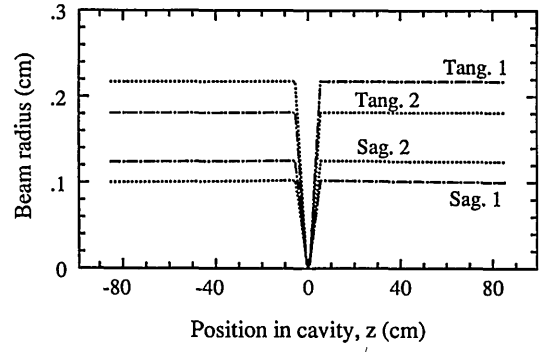

Fig. 3. Beam radii versus cavity position for two asymmetric TEM_{00} modes. Tangential and sagittal radii for modes 1 and 2 are indicated by Tang. 1, Sag. 1, Tang. 2, and Sag. 2. Calculations are based on the cavity shown in Fig. 2.

than for the mode-locked beam. When nonlinear $x-y$ coupling is omitted [by artificially setting $r(z) = 1$], calculations show a different result; the mode-locked sagittal beam is smaller than the cw sagittal beam on the left-hand side of the cavity.

Figure 2(a) shows the symmetric cavity, and Fig. 2(b) shows the tangential (x) radii for this cavity for an intracavity power of $P_{crit}/2$. When nonlinear $x-y$ coupling is included, calculations show the mode-locked tangential beam to be smaller than the cw tangential beam on both sides of the cavity. In this case, a vertical slit or the edge of a prism placed in the cavity would encourage mode locking. With nonlinear $x-y$ coupling omitted, however, the mode-locked tangential beam is larger than the cw tangential beam, so a slit would not encourage mode locking.

Although the two long arms of the cavities in the examples above are of equal length, the nonlinear coupling effect continues to be important for the case in which one of the arms is much longer than the other.

Nonlinear $x-y$ coupling is also needed for the accurate calculation of the shape, size, and position of the mode-locked beam in the gain medium. If these beam characteristics cause more pump power to be transferred to the mode-locked beam than to the cw beam, mode locking will tend to occur.

Figure 3 illustrates a qualitatively new feature brought about by nonlinear $x-y$ coupling—the existence of more than one TEM_{00} cavity mode. Using the symmetric Z cavity of Fig. 2 with an intracavity power of $P = 0.674P_{crit}$, we find two TEM_{00} solutions that are stable against perturbation, both asymmetric. Increasing the power slightly to $P = 0.726P_{crit}$, we find only solution stable against perturbation, this time symmetric. We have observed that multiple TEM_{00} cavity modes persist as we change cavity parameters such as the distances and angles between elements.

In summary, we show how to include the effect of nonlinear $x-y$ coupling arising from beam ellipticity in cavity calculations of mode-locked lasers. This effect, which is always present for noncircular beams, causes significant changes in beam radii and may result in multiple TEM_{00} cavity solutions.

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